Synthesis of a Helicopter Full-Authority Controller

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Nonlinear transformation theory is used to develop a full-authority trajectory controller for an autonomous helicopter. The transformation is simplified by the use of a forced singular perturbation. Overall system dynamics are separated into fast and slow reduced-order systems, each with nonlinear control. An analytical solution to inverting the slow time-scale dynamics is found through the inverse kinematics problem. The controller's performance is evaluated in terms of the controller's time-scale separation and uncertainties in the plant parameters.

Nomenclature

F_x, F_y, F_z	= aerodynamic forces in body frame
g	= acceleration of gravity
I_x, I_y, I_z, I_{xz}	= moments of inertia
L, M, N	= roll moment, pitch moment, yaw moment, respectively
m	= mass
p, q, r	= roll rate, pitch rate, yaw rate, respectively
U_i	= linear (pseudo) control
V_N, V_E, V_D	= inertial velocity components (north, east, down, respectively)
X_N, Y_E, Z_D	= inertial position coordinates (north, east, down, respectively)
α	= angle of attack
β	= sideslip
δ_a, δ_c, δ_e, δ_r	= lateral, collective, longitudinal, directional controls, respectively
ϕ, θ, ψ	= roll attitude, pitch attitude, yaw attitude, respectively

Subscripts

B	= body frame
I	= inertial frame
W	= wind frame

Introduction

CURRENT practice in helicopter flight control system design is to linearize the aircraft dynamics about several trim conditions and design linear perturbation controllers for various points within the envelope. Different controllers are necessary in various flight regimes to ensure an adequate response in all flight modes. Clearly, two of the drawbacks to this approach are the need to design more than one controller and the need to develop a gain schedule or mode switching scheme. Also, as the aircraft moves away from the design trim conditions, the flight control system performance degrades.

Using a nonlinear transformation, one can design a single linear time invariant controller that yields the desired response in all flight modes. This transformation is a mapping of states in the nonlinear system to states in a corresponding linear system, essentially a change of coordinates. The approach in this paper is based on a system with dynamics in block-triangular form.[1] States to be controlled are successively differentiated until the control terms appear in the equations. These differentiations form the mapping, i.e., the successive derivatives are the transformed states. A linear feedback control is designed in this linear space then transformed back into the corresponding nonlinear controls through an inverse of the mapping. This method has been applied to the design of a helicopter inertial flight-path trajectory controller.[2,3] However, in this earlier work, either the differentiations were carried out numerically or a linear approximation to the nonlinear system was made using a truncated Taylor series expansion.

The difficulty in position control lies in the fact that strict application of the differentiation technique leads one to using the force generating effect of the control surfaces to control the aircraft. Singh[4] points out that this is not the most practical way to use controls that are primarily moment generating devices. This concept is also found in controller design work by Hauser et al.[5] for an aircraft in which the small coupling between force and moment generation causes the system to be slightly nonminimum phase. Since the transformation is in effect a nonlinear pole-zero cancellation, it would be undesirable to try canceling a nonminimum phase system. In this study, the collective is used as a force control while the cyclic and directional controls are used as attitude controls.

With forced singular perturbation theory the order of the model can be reduced leading to simpler transformations.[6,7] The following sections of this paper demonstrate how singular perturbation theory simplifies the transformation over that in previous work.[2,3] Also presented is a method for solving the inverse kinematics problem that arises from this approach. Finally, the controller's performance is evaluated in terms of the controller time-scale separation and uncertainties in the plant parameters.

System

A six-degree-of-freedom helicopter model is used as the basis for the controller design. The inertial position and velocity dynamics are represented as follows:

$$\begin{bmatrix} \dot{X}_N \\ \dot{Y}_E \\ \dot{Z}_D \end{bmatrix} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = [L_{IB}] \begin{bmatrix} F_x/m \\ F_y/m \\ F_z/m \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{2}$$

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where $L_{IB}[-\phi, -\theta, -\psi]$ is the body-to-inertial frame transformation matrix. The dominant force control in Eq. (2) is the collective δ_c, which controls the normal force F_z. The attitude dynamics are represented by

$$\begin{bmatrix} \dot\theta \\ \dot\phi \\ \dot\psi \end{bmatrix} = \begin{bmatrix} q\cos\phi - r\sin\phi \\ p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \dot p \\ \dot q \\ \dot r \end{bmatrix} = \begin{bmatrix} (I_z\Sigma L + I_{xz}\Sigma N)/(I_xI_z - I_{xz}^2) \\ \Sigma M/I_y \\ I_{xz}\Sigma L + I_x\Sigma N)/(I_xI_z - I_{xz}^2) \end{bmatrix} \tag{4}$$

where the moment terms L, M, and N in Eq. (4) are largely functions of the lateral and longitudinal cyclic stick and directional controls δ_a, δ_e, and δ_r, respectively. One important assumption made in this model is that the cyclic stick and pedals are primarily moment generating controls and as such do not make a significant contribution to the body forces. Only collective is assumed to be a direct force control. This assumption has been justified for conventional aircraft[4] and is supported by the fast and slow time-scale controller approach, as outlined in the following sections.

Approach to the Transformation

In order for the preceding helicopter model dynamics to be controlled, the nonlinear system of equations must be mapped into a linear system of equations. As a refinement to an earlier approach,[2,3] singular perturbation theory is used to separate the full 12th-order system into two reduced-order systems based on slow and fast dynamics. This technique has been employed to control heading, flight-path angle, airspeed, and altitude in a high-speed fixed-wing aircraft.[6] In this study, the separation is made between the position dynamics, Eqs. (1) and (2), and the attitude dynamics, Eqs. (3) and (4). The idea is to control the position (X, Y, Z) in the slow time-scale system and the attitude (ϕ,θ,ψ) in the fast time scale.

Since the transformation technique requires the controlled states to be successively differentiated until control terms appear, the following three attitude dynamic equations are introduced by differentiating Eq. (3):

$$\ddot\theta = (\dot q - r\dot\phi)\cos\phi - (q\dot\phi + \dot r)\sin\phi$$

$$\ddot\phi = \dot p + (q\dot\phi + \dot r)\cos\phi\tan\theta + (\dot q - r\dot\phi)\sin\phi\tan\theta$$
$$+ (q\dot\theta\sin\phi + r\dot\theta\cos\phi)\sec^2\theta$$

$$\ddot\psi = [(\dot q - r\dot\phi)\sin\phi + (q\dot\phi + \dot r)\cos\phi]\sec\theta$$
$$+ \dot\theta(q\sin\phi + r\cos\phi)\tan\theta\sec\theta \tag{5}$$

where $\dot p,\dot q,\dot r$ represent Eqs. (4), which contain the control terms δ_a, δ_e, δ_r. Now the fast states are ϕ,θ,ψ and $\dot\phi,\dot\theta,\dot\psi$, and the slow are X_N, Y_E, Z_D and V_N, V_E, V_D.

Singular perturbation theory is a well-established technique for separating large dynamic systems into several reduced-order systems.[8,9] Kokotovic[9] has shown how large systems can be reduced to two smaller systems having slow and fast dynamics. The state equations can be written in the form

$$\dot x = f(x,z,u,\mu,t) \tag{6}$$

$$\mu\dot z = g(x,z,u,\mu,t) \tag{7}$$

where μ is a small parameter, x represents the slow time-scale dynamics, and z represents the fast dynamics. In the slow time scale, $\mu = 0$ and Eq. (7) becomes algebraic,

$$0 = g(\bar x,\bar z,\bar u,0,t) \tag{8}$$

where $\bar x,\bar z,\bar u$ are solutions to the approximate systems (6) and (8). The algebraic equation (8) can be solved for $\bar z$ as a function of $\bar x$ and $\bar u$:

$$\bar z = G(\bar x,\bar u,t) \tag{9}$$

substituting Eq. (9) into Eq. (6) yields

$$\dot{\bar x} = f[\bar x, G(\bar x,\bar u,t),\bar u,0,t] \tag{10}$$

$$\dot{\bar x} = \bar f(\bar x,\bar u,t) \tag{11}$$

In this study, a forced singular perturbation formulation is obtained by introducing a small parameter μ to the left-hand side of Eqs. (3) and (5). Setting μ to zero causes the fast dynamics to become algebraic, implying that the dynamics respond instantaneously in the slow time scale. This results in the slow time-scale problem in which the dynamic equations for the body attitudes are replaced by algebraic equations, i.e., $\dot{\bar\phi} = \dot{\bar\theta} = \dot{\bar\psi} = 0$ and $\ddot{\bar\phi} = \ddot{\bar\theta} = \ddot{\bar\psi} = 0$. In the algebraic equations, $\bar p,\bar q,\bar r$ and $\bar\phi,\bar\theta,\bar\psi$ are used to represent the slow time-scale values for the body rates and attitudes. The interpretation of this simplification is that in the slow time scale the fast states have already reached their steady-state values.

Because of the block-triangular structure of the equations of motion, the only control to appear in the slow dynamics equations (1) and (2) is the collective. The other controls for the slow dynamics are the fast states, $\bar\phi,\bar\theta,\bar\psi$, which appear "control-like" and are related to the actual controls through the algebraic equations. Therefore, the position dynamics are controlled through the body attitude and the collective. Specifically, the aircraft will be controlled by using collective to adjust the magnitude of the thrust vector and by using the body attitude to orient the thrust vector in the desired direction. Since the vehicle is in moment equilibrium in the slow time scale, moments generated by the collective are trimmed out by the cyclic and tail rotor. It is interesting to note that this type of scheme is usually employed in classical flight control system design.

In the fast time scale, the slow states are assumed to be known values. An inner loop is designed to maintain the actual body attitude ϕ,θ,ψ, as close to the slow time-scale commanded values $\bar\phi,\bar\theta,\bar\psi$, as possible. This is implemented by defining three error states

$$\Delta\theta = \theta - \bar\theta$$
$$\Delta\phi = \phi - \bar\phi$$
$$\Delta\psi = \psi - \bar\psi \tag{12}$$

Since $\dot{\bar\phi} = \dot{\bar\theta} = \dot{\bar\psi} = 0$ and $\ddot{\bar\phi} = \ddot{\bar\theta} = \ddot{\bar\psi} = 0$, the dynamic equations for the error states are the same as Eqs. (3) and (5).

One can see that the time-scale separation immediately provides the transformation through two successive differentiations. In the linear system, the states are $\Delta\phi,\Delta\theta,\Delta\psi,\Delta\dot\phi,\Delta\dot\theta,\Delta\dot\psi$ and X_N, Y_E, Z_D, V_N, V_E, V_D. The linear or pseudocontrol variables are

$$U_1 = \dot V_N, \qquad U_4 = \Delta\ddot\theta$$
$$U_2 = \dot V_E, \qquad U_5 = \Delta\ddot\phi$$
$$U_3 = \dot V_D, \qquad U_6 = \Delta\ddot\psi$$

The resulting linear system is in Brunovsky's canonical form.[2]

Figure 1 shows a block diagram of the two time-scale controller. The slow time-scale system is designed to track the $X_N(t)$, $Y_E(t)$, $Z_D(t)$ position coordinates through attitude commands $(\bar\phi,\bar\theta,\bar\psi)$ and the collective. The fast time-scale system tracks the attitude commands of the slow system using pitch and roll cyclic and yaw pedals. Figures 2a and 2b give a

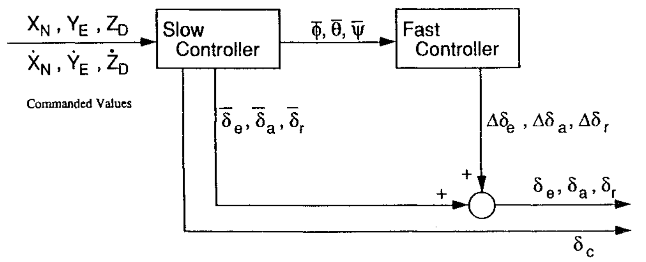

Fig. 1 Control system block diagram.

detailed view of the slow and fast systems to show the relationship of the transformation map to the nonlinear system and the linear feedback control. In the design of the feedback controller, note that the path from the inverse of the mapping through the nonlinear dynamics and the mapping itself is linear. Therefore, to the feedback controller, the nonlinear system appears linear. A constant feedback gain matrix can be chosen to yield the desired linear response characteristics.

Inverse Transformation

An inverse of the transformation must be found so that the nonlinear controls can be determined from the linear feedback controls. In the slow time scale the linear feedback controls U_1, U_2, U_3 are proportional to the position errors. The nonlinear controls $\bar{\phi}, \bar{\theta}, \bar{\psi}$ (and $\bar{\delta}_a, \bar{\delta}_e, \bar{\delta}_r$) must be found from an inverse transformation of the linear controls. Similarly, in the fast time scale, the nonlinear controls $\Delta\delta_a$, $\Delta\delta_e$, $\Delta\delta_r$ are determined from the linear attitude feedback controls, U_4, U_5, U_6.

First the slow time scale is examined. Here, body Euler angles in Eq. (2) are represented by their steady-state values $\bar{\phi}, \bar{\theta}, \bar{\psi}$. Rearranging Eq. (2) as

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 - g \end{bmatrix} = \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D - g \end{bmatrix} = [L_{IB}] \begin{bmatrix} F_x/m \\ F_y/m \\ F_z/m \end{bmatrix} \tag{13}$$

and defining $\tilde{U}_3 = U_3 - g$ yields a set of three nonlinear equations. The linear control vector U is known from feedback and the body forces F_x and F_y are found from the aerodynamic model (or from sensed accelerations on an actual aircraft). Since L_{IB} is purely a rotation transformation, the magnitudes of the control vector U and the body force vector F/m must be equal in order to find a solution for the Euler angles. The collective was used to adjust the magnitude of the body forces because it is a direct force generating control and appears in the slow time-scale dynamics in F_z:

$$\|F/m\| = \|U\| \tag{14}$$

$$F_z(\delta_c) = \pm m\sqrt{U^2 - (F_x^2 + F_y^2)/m^2} \tag{15}$$

The sign on the square root is chosen to be the same as that of $\tilde{U}_3$. Solving Eq. (15) for δ_c yields a nonlinear feedback control law for the collective.

With the vector magnitude constraint satisfied, the remaining task is to find a solution to the set of simultaneous nonlinear equations. Solving Eq. (13) for $\bar{\phi}, \bar{\theta}, \bar{\psi}$ is the inverse kinematics problem because the two force vectors are known and the rotation matrix must be found—the inverse of the normal problem in flight mechanics. A method is presented in the next section on solving this problem.

The fast time-scale control system is designed to be a regulator minimizing the attitude error states $\Delta\phi$, $\Delta\theta$, $\Delta\psi$. With the transformation, the pseudocontrols become

$$U_4 = \Delta\ddot{\phi} = \ddot{\phi}$$

$$U_5 = \Delta\ddot{\theta} = \ddot{\theta}$$

$$U_6 = \Delta\ddot{\psi} = \ddot{\psi} \tag{16}$$

Writing the angular accelerations in terms of the derivatives of the Euler angles and substituting into Eqs. (16) yields,

$$\dot{p} = U_4 - U_6 \sin\theta - \dot{\psi}\dot{\theta}\cos\theta$$

$$\dot{q} = U_5 \cos\phi - \dot{\theta}\dot{\phi}\sin\phi + U_6 \sin\phi\cos\theta$$

$$+ \dot{\psi}\dot{\phi}\cos\phi\cos\theta - \dot{\psi}\dot{\theta}\sin\phi\sin\theta$$

$$\dot{r} = -U_5 \sin\phi - \dot{\theta}\dot{\phi}\cos\phi + U_6\cos\phi\cos\theta$$

$$- \dot{\psi}\dot{\phi}\sin\phi\cos\theta - \dot{\psi}\dot{\theta}\cos\phi\sin\theta \tag{17}$$

where $\dot{p}, \dot{q}, \dot{r}$ are functions of the cyclic stick and pedals described in Eq. (4). Equations (17) are solved simultaneously to determine the total cyclic stick and pedal commands.

Solution to the Inverse Kinematics Problem

In the previous section it was pointed out that determining $\bar{\phi}, \bar{\theta}, \bar{\psi}$ from $U_1, U_2, \tilde{U}_3$ requires solving the inverse kinematics

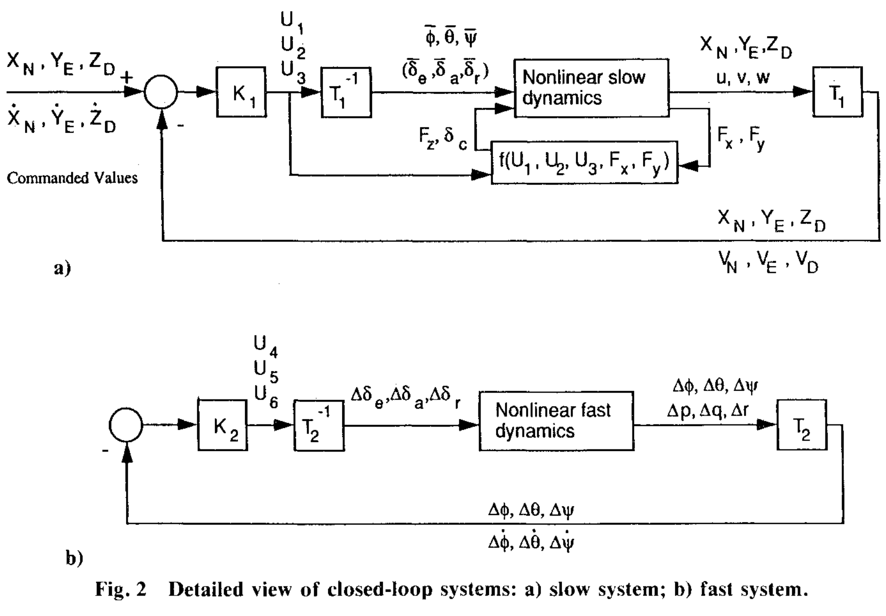

Fig. 2 Detailed view of closed-loop systems: a) slow system; b) fast system.

problem.[10] First, both the control vector U, and the body force vector F/m are transformed to the wind frame

$$\begin{bmatrix} U_1 \\ U_2 \\ \bar{U}_3 \end{bmatrix}_W = [L_{WI}] \begin{bmatrix} U_1 \\ U_2 \\ \bar{U}_3 \end{bmatrix}_I \tag{18}$$

where $L_{WI}(\psi_W, \theta_W, \phi_W)$ is the inertial-to-wind transformation matrix.

Equation (13) implies that

$$\begin{bmatrix} U_1 \\ U_2 \\ \bar{U}_3 \end{bmatrix}_W = [L_{WB}(0, -\alpha, \beta)] \begin{bmatrix} F_x/m \\ F_y/m \\ F_z/m \end{bmatrix}_B \tag{19}$$

Equating the three control components from Eqs. (18) and (19) yields the following:

$$U_1 \cos\theta_W \cos\psi_W + U_2 \cos\theta_W \sin\psi_W - \bar{U}_3 \sin\theta_W$$
$$= (F_x/m)\cos\alpha\cos\beta + (F_y/m)\sin\beta$$
$$+ (F_z/m)\sin\alpha\cos\beta \tag{20}$$

$$U_1(\sin\phi_W \sin\theta_W \cos\psi_W - \cos\phi_W \sin\psi_W)$$
$$+ U_2(\sin\phi_W \sin\theta_W \sin\psi_W + \cos\phi_W \cos\psi_W)$$
$$+ \bar{U}_3(\sin\phi_W \cos\theta_W) = (-F_x/m)\cos\alpha\sin\beta$$
$$+ (F_y/m)\cos\beta - (F_z/m)\sin\alpha\sin\beta \tag{21}$$

At this point, Eqs. (20) and (21) contain five unknowns, θ_W, ψ_W, ϕ_W, β, and α. If the atmosphere is assumed to be at rest, the commanded inertial velocity components can be related to the airspeed V_T, heading ψ_W, and flight-path angle θ_W as follows:

$$V_T = \sqrt{\dot{X}_c^2 + \dot{Y}_c^2 + \dot{Z}_c^2}$$
$$\theta_W = \sin^{-1}(-\dot{Z}_c/V_T) \tag{22}$$
$$\psi_W = \tan^{-1}(\dot{Y}_c/\dot{X}_c)$$

It is assumed that the helicopter must track the trajectories with zero sideslip, i.e., $\beta = 0$. Now, with three of the angles known, Eq. (20) reduces to a transcendental one in α and equation Eq. (21) reduces to one in ϕ_W. Both can be solved by a simple iteration scheme.

The desired attitude of the aircraft can now be found by using the fact that

$$L_{BI} = L_{BW} L_{WI} \tag{23}$$

and equating individual elements. The solutions are the commanded attitudes $\bar{\phi}$, $\bar{\theta}$, $\bar{\psi}$ for the slow time-scale system. It is the task of the fast controller to make the actual body attitude angles track these commanded values.

Transformed Linear Feedback Controller

The main advantage to the nonlinear transformation approach is that the feedback loop is closed as a linear feedback system. In this case, the linear system consists of three sets of double integrators for each time-scale system. Each string corresponds to an open-loop system having two poles at the origin. A simple proportional plus derivative feedback design was selected for the linear controller. The slow time-scale controller has the derivative of the position command as a feed-forward signal. This was done to eliminate the steady-state error associated with a second-order system subject to a ramp command. The transfer function for the position closed-loop systems is

$$G(s)_{cl} = \frac{K_d s + K_p}{s^2 + K_d s + K_p} \tag{24}$$

For the body attitude dynamics, the closed-loop system is

$$G(s)_{cl} = \frac{K_p}{s^2 + K_d s + K_p} \tag{25}$$

K_d and K_p can be determined for a given damping ratio and natural frequency or for a given set of eigenvalues. The choice of a suitable eigenvalue arrangement is somewhat arbitrary; however, some guidelines can be used. For instance, one would not want to design a body attitude controller that could excite higher modes such as the rotor flapping and lead-lag modes or actuator dynamics. Also, adequate time-scale separation or eigenvalue separation between the slow and fast systems is required, as shown in the next section. For this study, the slow time-scale eigenvalues were chosen as $-1 \pm 1i$ and the fast time scale as $-2 \pm 2i$. This gives a 0.7 damping ratio for both systems and a natural frequency of 1.4 rad/s for the slow system and 2.8 rad/s for the fast system. Solving for the gains on each system yields $K_d = 2$ and $K_p = 2$ for the slow time-scale controller and $K_d = 4$ and $K_p = 8$ for the fast time-scale controller.

Controller Evaluation

For the controller design evaluation, the TMAN program developed at NASA Ames for nap of the earth one-on-one air combat simulation was chosen.[11] TMAN is a rather simple, nonlinear rigid-body model with quasistatic linearized aerodynamics, a simplified closed-form trim calculation, and first-order engine dynamics. The body forces are represented as

$$\begin{bmatrix} F_x/m \\ F_y/m \\ F_z/m \end{bmatrix} = \begin{bmatrix} X_u u \\ Y_v v \\ \ddot{Z}_{trim} + Z_w w + Z_{\delta_c}\delta_c \end{bmatrix} \tag{26}$$

where $\ddot{Z}_{trim} = -Z_w w_{trim} - g\cos\theta_{trim}$. Using Eqs. (26) in Eq. (15) gives the nonlinear feedback control law for the collective as

$$\delta_c = \frac{-\ddot{Z}_{trim} - Z_w w \pm \sqrt{U_1^2 + U_2^2 + \bar{U}_3^2 - (F_x/m)^2 - (F_y/m)^2}}{Z_{\delta_c}} \tag{27}$$

Angular acceleration in TMAN is defined as follows:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L_p p + L_v v + L_{\delta_a}\delta_a \\ M_q q + M_u u + M_{\delta_e}\delta_e \\ N_r r + N_v v + N_p p + N_{\delta_r}\delta_r \end{bmatrix} \tag{28}$$

Substituting Eq. (28) into Eq. (17) transforms the linear feedback controls into corresponding cyclic stick and pedal commands:

$$\delta_a = [U_4 - (U_6)\sin\theta - \dot{\psi}\dot{\theta}\cos\theta - L_p p - L_v v]/L_{\delta_a}$$
$$\delta_e = (U_5 \cos\phi - \dot{\theta}\dot{\phi}\sin\phi + U_6 \sin\phi\cos\theta + \dot{\psi}\dot{\phi}\cos\phi\cos\theta$$
$$- \dot{\psi}\dot{\theta}\sin\phi\sin\theta - M_u u - M_q q)/M_{\delta_e}$$
$$\delta_r = (-U_5 \sin\phi - \dot{\theta}\dot{\phi}\cos\phi + U_6\cos\phi\cos\theta$$
$$- \dot{\psi}\dot{\phi}\sin\phi\cos\theta - \dot{\psi}\dot{\theta}\cos\phi\sin\theta - N_r r - N_p p$$
$$- N_v v)/N_{\delta_r} \tag{29}$$

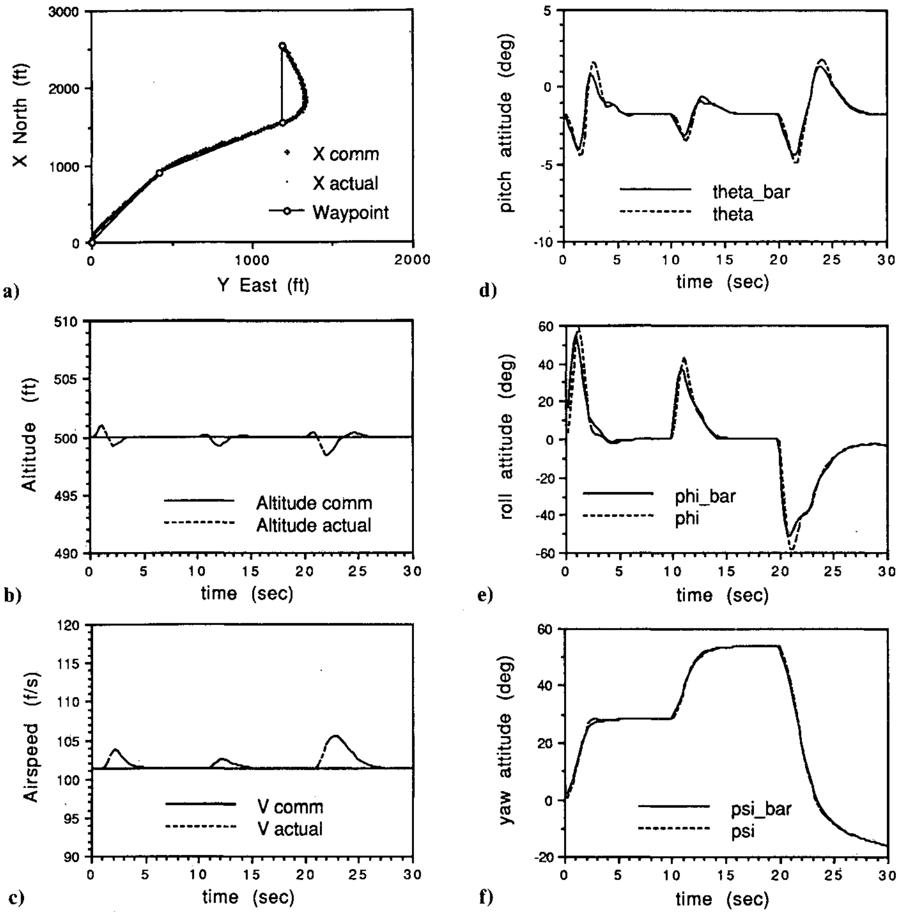

Fig. 3 Slalom maneuver: a) ground track; b) altitude response; c) airspeed response; d) pitch response; e) roll response; f) yaw response.

A constant altitude, constant airspeed slalom maneuver was flown to demonstrate the tracking performance of the controller. Figure 3a shows a ground track laid out by way points, a commanded flight path created by a smoothing function, and the actual flight path. One can see that the controller tracks the commanded trajectory quite well. In addition, Figs. 3b and 3c show that altitude and airspeed are well maintained. Figures 3d–3f give a comparison of the slow time-scale commanded body attitudes and the actual attitudes of the aircraft. Note that the pitch attitude is tracked to within 1 deg and the bank angle to within 5 deg.

Time-Scale Separation

To examine the effect of time-scale separation on the controller's performance, a slowed fast controller was made by using the same gains in both the fast and slow time-scale controllers. The poles for both controllers were placed together at $-1 \pm 1i$. Figure 4 shows a comparison between the nominal system (Fig. 4a) and the slowed fast controller (Fig. 4b) for a step heading change. Here, the lagging attitude response interferes with the commanded attitude solution from the slow time-scale controller. The result is that the slow controller's attitude solution is now a slowly decaying oscillation. Hence, these results underscore the need to design the two time-scale controllers with significant eigenvalue separation.

Parameter Uncertainties

The effects of uncertainties in the plant parameters were evaluated by linearizing the closed-loop nonlinear system using a numerical central difference scheme. Recall that when the mapping is exact the transformation takes states into an invariant linear system. Numerical linearizations at hover and 200 kt verified that this is indeed the case when no errors exist between the mapping equations and the dynamic equations used in the simulation. In both instances, the slow time-scale system yielded eigenvalues at $-1 \pm 1i$, whereas the fast time-scale system gave eigenvalues at $-2 \pm 2i$.

By introducing scale factors in the simulation's dynamic equations, errors between the dynamics and the map can be created. For example, in the equation for calculating the body Z force for the simulation, the control derivative Z_{δ_c} is multiplied by some percentage change. At the same time, however, the percentage change in the Z force equation used in the

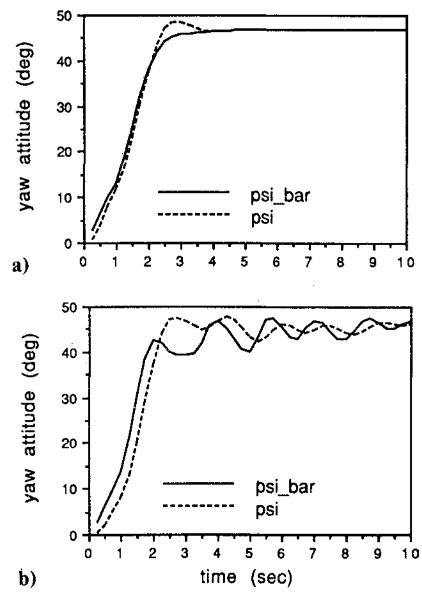

Fig. 4 Comparison of yaw response between a) nominal design; b) slowed fast controller.

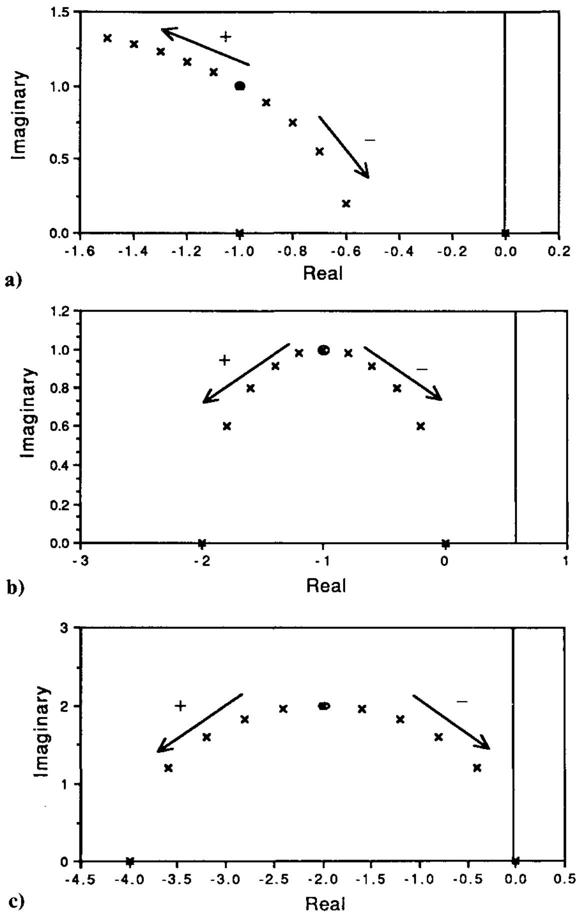

Fig. 5 Pole migration due to modeling errors: a) Z_{δ_c} coefficient; b) total Z force; c) total rolling moment.

mapping was not included. Hence, some error is introduced between the dynamic equations and the mapping. Figure 5a shows the effect of such errors in the eigenvalues of the slow time-scale system. In this case, Z_{δ_c} was multiplied by a scale factor ranging from -100% to $+100\%$. The pole migration is marked off in 20% intervals with the positive direction indicating that the value of the parameter used in the simulation was greater than the value used in the mapping. This case was run near hover with the body and inertial axes aligned; therefore, only one pair of eigenvalues is affected. Note that the effect of a 20% error in Z_{δ_c} is small. Similar effects occur when error is introduced into any of the other stability derivative terms. Much larger changes occur if the total body forces, including the gravity components, are varied -100%–$+100\%$. Figure 5b shows that the pair of poles migrates to the real axis as the total Z force error increases. The result is identical for variations in the longitudinal and side forces.

Results for errors in the fast time-scale dynamics of the total rolling moment are shown in Fig. 5c. In this instance, the poles migrate from the nominal position at $-2 \pm 2i$ to the real axis. It would appear that the controller should be able to easily tolerate 20% error in the total force and moment calculations used in the mapping. This gives an indication of the relative robustness of the flight control loop to plant parameter variations. It is also worth noting that using the nonlinear mapping for linearization is an alternative to using stability derivatives. In this sense, a linear control methodology that reduces sensitivity to pole migration can be used to increase the robustness of the system.

Conclusions

Nonlinear transformation theory has been used to design a full-authority inertial position controller for a conventional helicopter. A forced singular perturbation method was presented for generating two reduced-order systems that allows for the mapping to be determined analytically. In a new application of this technique, the separation was made between the inertial position and attitude dynamics. Unique to this study is the control law development for the collective in conjunction with a solution to the inverse kinematics problem to carry out the inverse mapping. The collective is used as a force control to adjust the magnitude of the thrust vector while the body attitude is used to orient it in the desired direction. The controller demonstrated excellent tracking with the time-scale separation chosen for this study. It was shown that, had the time-scale separation been less, the tracking performance would have been degraded. An initial investigation into the effects of uncertainty in the plant parameters indicated that the controller should still function properly with 20% error in the total force and moment calculations used in the transformation.

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